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Thermodynamics of Brittle Fracture

by Michael A. Grinfeld and Tim W. Wright

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14. ABSTRACT

This report outlines the results of the effort to suggest a theoretical method of describing brittle ceramic armor. We analyze thermodynamics and kinetics of brittle fracture by combining the Gibbs approach to the study of heterogeneous thermodynamic systems, the Griffith energetic criterion for crack growth, and the Onsager irreversible thermodynamics. The central object of the study is the exchange between accumulated elastic energy and “chemical” energy accumulated in cohesive bonds of material particles. One of the main thrusts of the research is development of a continuum model describing dwell-defeat transition for a metallic projectile hitting ceramic armor. As compared with known theories of fracture, we describe the morphology of the damaged zone. This makes our approach somewhat similar to the Gibbs theory of heterogeneous systems.

15. SUBJECT TERMS

ceramic armor, brittle fracture, failure waves, morphological instability

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1. Introduction

Brittle materials (i.e., ceramics) are being considered for an increasing number of applications ranging from armor elements to gun tube inserts to engine components, where they must withstand intense dynamic loads. The damage and massive failure processes that can occur in these conditions are not well understood. Because ceramics typically have lower symmetry than most metals, the basic crystal components are not only highly anisotropic but also have only limited slip systems for plastic deformation in polycrystalline configurations. Therefore, they must rely on other mechanisms such as twinning, microcracking, and void formation, as well as limited plasticity, to achieve required deformation. Adjacent grains of differing orientation will find it hard to maintain coherent boundaries; large stress concentrations within grains may build up; and large oscillations in stress and strain may occur from grain to grain. Homogenization theories can predict average material properties. However, in the circumstances just described, knowledge of the oscillations is also necessary. It is the purpose of this effort to develop techniques that will give an account of the oscillations, as well as the averages, and to apply the resulting theory to several important topics such as damage, incipient microcracking, and fracture waves.

During several of the last decades, ceramic-based armor technologies and corresponding experiments and theories have shown a remarkable progress. One of the brightest achievements of the last decade is the discovery of the phenomena of dwell-penetration transition and of a defeat of a projectile targeting ceramic armor (*1–3*). The phenomenon of a projectile defeat is accompanied by the appearance of an intensively fractured zone (IFZ) within the bulk of a ceramic target. The shape of this zone resembles a tri-axial ellipsoid (see figure 1, where the IFZ zone is shown in white). In their recent publication, LaSalvia and Normandia (*4*) interpreted it on the basis of the famous Bussinesq solution of classical theory of elasticity. Similar observations of IFZ are known in ceramic indentation experiments (*5*) and in the seismology of earthquake epicenters (*6*).

When dealing with IFZ, it is often desirable to extend the information about the overall modulae with the information about the shape and morphology of the IFZ. Such information can be obtained by using additional, specific criteria or more general principles of mechanics or thermodynamics. In this report, following the minimum energy approaches of Gibbs (*7–8*) and Griffith (*9*), we propose one such thermodynamic approach, which allows one to determine the shape of the IFZ simultaneously with the distributions of stresses and strains within the systems of interest. The general methods and results of the suggested approach are illustrated by considering an instructive problem of nucleation of a damaged zone in an isotropic solid space under sufficiently high uniform stresses at infinity.

Source: Picture courtesy of J. LaSalvia and M. Normandia (*4*).

Figure 1. Damaged observation from recovered ceramics.

2. The Substance Model

Consider a macroscopically heterogeneous configuration possessing two macrodomains, ω_+ and ω_-, occupied by the consolidated and damaged modifications, respectively. Let γ be the interface separating the two domains and S the external boundary of the system (see figure 2).

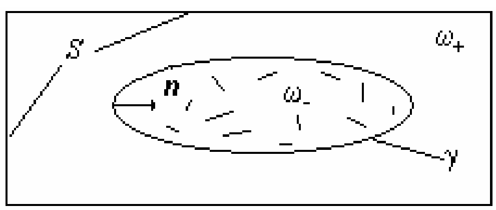

Figure 2. The geometry of the system.

The consolidated (intact) modification is characterized by the elastic energy density per unit volume

$$e_+(u_{i|j}) = \frac{1}{2} c_+^{ijkl} u_{i|j} u_{k|l} \,, \tag{1}$$

where $u_i(x)$ is the displacement at the point x^i $(u_{i|j} = \partial u_i / \partial x^j)$ and c_+^{ijkl} is the elasticity tensor of the consolidated modification. The absence of the free-constant and linear terms in the Taylor expansion of $e(u_{i|j})$ indicates that the configuration with $u_i = 0$ is stress free, and it is chosen as

the base for the calculation of the elastic energy of the substance. For the damaged modification of our system we can use an "effective" elastic energy density as follows:

$$e_{-}(u_{i|j}) = e^{\circ} + \frac{1}{2} c_{-}^{ijkl} u_{i|j} u_{k|l}. \tag{2}$$

The damaged modification is elastically weaker than the intact one, i.e., its elasticities in some sense are smaller than the elasticities of the intact modification. Besides, there is a nonzero constant e° in the elastic energy density of the damage modification—this constant reflects the work required to produce defects in the intact modification. Schematically, the two energies are shown in figure 3.

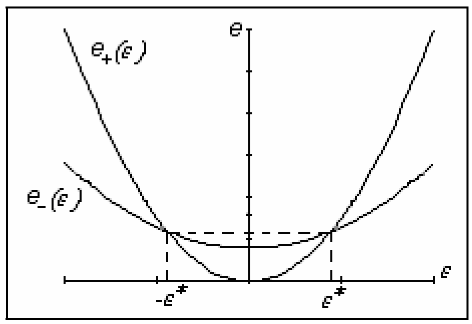

Figure 3. The specific energy density of the intact (e_{+}) and damaged (e_{-}) modifications.

The parabola of the consolidated substance passes through the origin and has the fastest growth with ε. It is clear that for $|\varepsilon| < |\varepsilon^{*}|$, the elastic energy density of the consolidated phase is less than in the damaged phase (for $|\varepsilon| > |\varepsilon^{*}|$, the elastic energy density of consolidated phase is greater than in the damaged phase). The trade-off between the opposite effects of the damage on "free-stress" energy e_{-} and on the elasticity tensor c_{-}^{ijkl} at various deformations is the central conflict under study with the help of our suggested model.

The simplest model of the previously mentioned type is the model of an isotropic two-state elastic substance. For this model, the energy densities for two modifications are given by the following formula:

$$e_{\pm}(u_{i|j}) = \kappa_{\pm} + \mu_{\pm}\left(\frac{\nu_{\pm}}{1-2\nu_{\pm}} u^{i}_{.|i} u^{j}_{.|j} + u_{(i|j)} u^{ij}_{.|.}\right), \tag{3}$$

where $\mu_{\pm}$ are the shear Lame modules and $\nu_{\pm}$ are the Poisson's ratios ($\kappa_{+} = 0$).

3. Continuum Model of Fracture

Per classification of Grinfeld and Wright (*10, 11*), the model just considered is a certain two-state discrete fracture model. A continuum fracture model should include a damage parameter κ that can assume a continuum range of values. Corresponding elastic energy e function is becoming a function of the elastic gradients $u_{i|j}$ and of the damage parameter κ: $e = e(u_{i|j}, \kappa)$. For such a model, the associated equilibrium master system includes the following two bulk equations:

$$\frac{\partial}{\partial x^j}\frac{\partial e(u_{ni|n}, \kappa)}{\partial u_{i|j}} = 0, \tag{4}$$

and

$$\frac{\partial e(u_{i|j}, \kappa)}{\partial \kappa} = 0\,. \tag{5}$$

The associated system of quasistatic evolution reads as follows:

$$\frac{\partial}{\partial x^j}\frac{\partial e(u_{ni|n}, \kappa)}{\partial u_{i|j}} = 0, \tag{6}$$

and

$$\frac{\partial \kappa}{\partial t} = -K\frac{\partial e(u_{i|j}, \kappa)}{\partial \kappa}, \tag{7}$$

where K is a certain positive kinetic constant or function.

The associated dynamic system reads as follows:

$$\rho\frac{\partial^2 u^i}{\partial t^2} = \frac{\partial}{\partial x^j}\frac{\partial e(u_{ni|n}, \kappa)}{\partial u_{i|j}}, \tag{8}$$

and

$$\frac{\partial \kappa}{\partial t} = -K\frac{\partial e(u_{i|j}, \kappa)}{\partial \kappa}. \tag{9}$$

The important issue here is the choice of an appropriate energy function $e = e(u_{i|j}, \kappa)$ and the kinetics function *K*. The proper choice is a hard problem of physics of ceramic materials. Some insight can be acquired by using approaches accepted in phenomenological damage theory (*12*)

and the theories of proliferation of vacancies in ceramics sintering (*13*). Needless to say, cracking is not a proliferation of vacancies because of various reasons, but this might be a good first step.

The quasistatic system presented in equations 6 and 7 was used to numerically explore the behavior of a circular plate with a thin elliptic cavity under action of external pressure (figure 4). The kinetics function K was assumed a certain positive constant. For the energy density, we have chosen the following Kachanov-Lifshitz function:

$$e(u_{i|j},\kappa) = \mu\varphi(\kappa)\left(\frac{\nu}{1-2\nu}u^{i}_{.|i}u^{j}_{.|j} + u_{(i|j)}u^{ij}_{.|.}\right) + \frac{\xi}{2}\left(\kappa-\kappa^{\circ}\right)^2, \tag{10}$$

with the damage function $\phi(\kappa)$ (*10, 11*) as follows:

$$\phi(\kappa) = 1-\left(1-c_{\min}\right)\frac{\kappa}{\kappa^*}, \tag{11}$$

$$0 \le \kappa \le \kappa^*, \tag{12}$$

and

$$0 < c_{\min} \le 1. \tag{13}$$

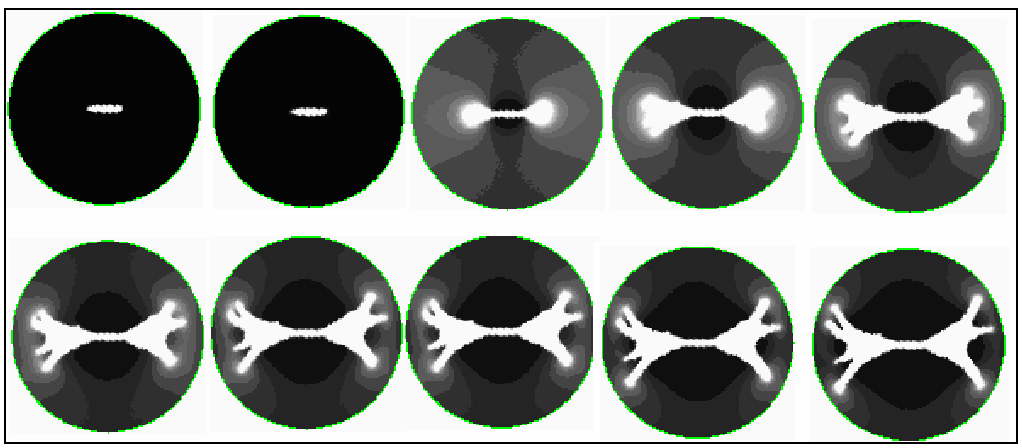

Figure 4. Development of damage in a circular elastic plate with a cavity.

Thus, the suggested model depends on six constants—μ, ν, ξ, κ°, κ^*, and $c_{\min}$. The first two of them are just the shear modulus and Poisson's ratio of the intact substance; the physical meaning of the remaining four constants are explained elsewhere. For a certain choice of the constants, the evolution of the relative damage parameter κ/κ^* is shown in figure 4. The depth of gray reflects the magnitude of the relative damage parameter, white corresponds to a fully damaged material, and black to a fully undamaged. Not surprisingly, the largest damage occurs at the tips of the cavity.

4. Summary and Conclusions

- The proposed theoretical approach to brittle fracture allows the determination of the morphology of damaged zones together with the traditional fields like deformation and stress.
- Similar to the Griffith model, the proposed approach is based on the trade-off between the elastic energy and the energy of chemical bonds.
- In the proposed approach, the damaged and intact substances are treated as different phases of the same substance, and "phase transformation" damage-recovery is taken into account.
- The equilibrium system includes coupled equations for the unknown displacement field, damage parameter, and morphology.
- Because of its deep nonlinearity, even for small deformations, the equilibrium system may have multiple solutions.
- The equilibrium system does not allow static solution for some boundary loads; hence, the system should be extended by the inclusion of inertia and dissipation mechanisms.

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